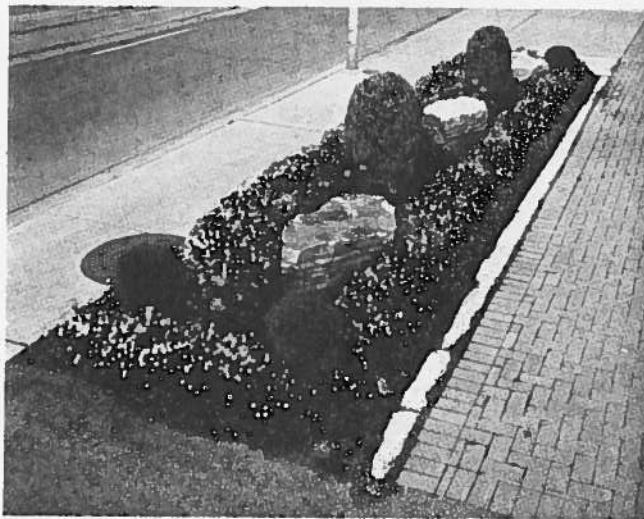
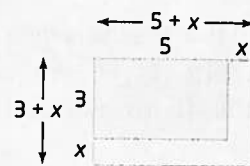


# 5.1

## Multiply Polynomials



A rectangular garden measures 3 m by 5 m. If each dimension is increased by the same amount to expand the garden, how can you model the area of the new garden using a polynomial?



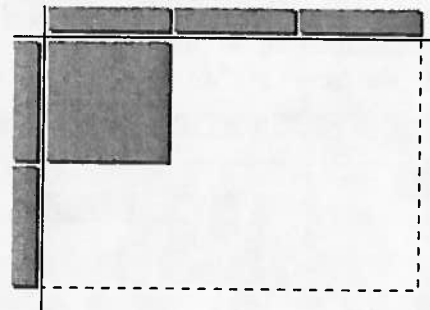
**Tools**  
 ■ algebra tiles

### Investigate A

How can you model the multiplication of polynomials?

Method 1: Use Algebra Tiles

1. To show the product  $(2x)(3x)$ , use algebra tiles to model  $2x$  and  $3x$  as the dimensions of a rectangle. Then, fill in the rectangle with tiles to find an expression for the area. An  $x^2$ -tile has been placed to begin the process. How many  $x^2$ -tiles are needed to fill in the rectangle? What is the area of the rectangle? What is the product  $(2x)(3x)$ ?

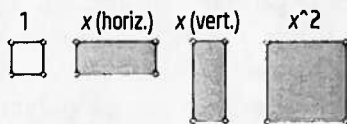


2. To show the product  $(2x)(3x + 3)$ , use algebra tiles to model  $2x$  and  $3x + 3$  as the dimensions of a rectangle.
  - a) Use two  $x$ -tiles to form one dimension along the left side. Use three  $x$ -tiles and three unit tiles to form the other dimension along the top. Complete the area of the rectangle.
  - b) How many  $x^2$ -tiles and  $x$ -tiles are needed? What is the resulting product?

3. Use algebra tiles to model the product  $(3x)(2x + 1)$ .  
What is the resulting product?
4. Use the same process to model the product of two binomials.  
What is each resulting product?
  - a)  $(x + 1)(x + 2)$       b)  $(x + 2)(x + 4)$
  - c)  $(x + 3)(2x + 1)$       d)  $(2x + 3)(x + 1)$
5. **Reflect** Consider your results from step 4.
  - a) Describe how you can use algebra tiles to multiply two binomials.
  - b) How are the terms in the resulting products related to the terms of the two binomials? Write a general rule for multiplying two binomials.
6. Use your rule to find each product.
  - a)  $(x + 3)(x + 8)$       b)  $(2x + 5)(x + 4)$
  - c)  $(4x + 7)(3x + 1)$

**Method 2: Use The Geometer's Sketchpad®**

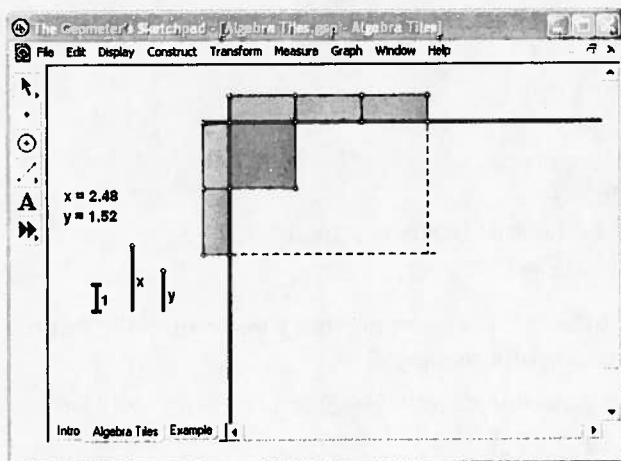
For this activity, you will use unit tiles, x-tiles (horizontal and vertical), and  $x^2$ -tiles.



**Tools**

- computer with The Geometer's Sketchpad®
- Algebra Tiles.gsp

1. To show the product  $(2x)(3x)$ , use virtual algebra tiles to model  $2x$  and  $3x$  as the dimensions of a rectangle. Then, fill in the rectangle with tiles to find an expression for the area.
  - Click and hold the **Custom Tool** icon. Select **x (vertical)** and place two tiles along the vertical line, as shown.
  - Repeat for **x (horizontal)**, placing three tiles along the horizontal line, as shown.
  - Repeat for  **$x^2$**  ( $x^2$ -tile) to complete the area of the rectangle.



How many  $x^2$ -tiles are needed? What is the area of the rectangle?  
What is the product  $(2x)(3x)$ ?

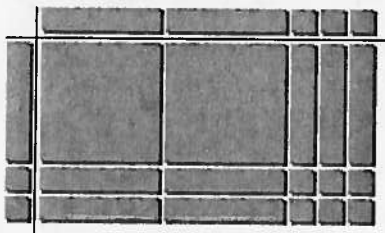
## Example 1 Model a Binomial Product

Model the binomial product  $(x + 2)(2x + 3)$ .

### Solution

#### Method 1: Use Algebra Tiles

Use algebra tiles to create a rectangle with width  $x + 2$  and length  $2x + 3$ .



There are two  $x^2$ -tiles, seven  $x$ -tiles, and six unit tiles.

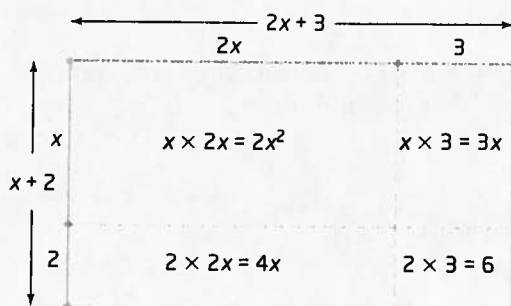
The area of the rectangle is  $2x^2 + 7x + 6$ .  
 $(x + 2)(2x + 3) = 2x^2 + 7x + 6$

#### Method 2: Use a Diagram

Draw a vertical segment of any length and label it  $x + 2$  units. Divide the segment into a section  $x$  units long and a section 2 units long.

Perpendicular to the top of the vertical segment, draw a horizontal segment,  $2x + 3$  units long, divided into a section  $2x$  units long and a section 3 units long.

Complete the rectangle by drawing sides opposite  $x + 2$  and  $2x + 3$ . Then, draw horizontal and vertical dashed segments from the section marks to the opposite sides, as shown. Find the areas of the four sections that make up the whole rectangle.



The binomial product  $(x + 2)(2x + 3)$  equals the sum of the four areas.  
 $(x + 2)(2x + 3) = 2x^2 + 3x + 4x + 6$   
 $= 2x^2 + 7x + 6$

## Example 2 Use the Distributive Property

Find each binomial product.

- a)  $(x + 2)(x + 5)$
- b)  $(x - 2)(x + 4)$
- c)  $(3x + 7)(x - 5)$

### Solution

- a) To multiply two binomials, use the distributive property. Then, simplify by collecting like terms.

$$\begin{aligned}(x + 2)(x + 5) &= x(x + 5) + 2(x + 5) \\ &= x(x) + x(5) + 2(x) + 2(5) \\ &= x^2 + 5x + 2x + 10 \\ &= x^2 + 7x + 10\end{aligned}$$

b)  $(x - 2)(x + 4)$

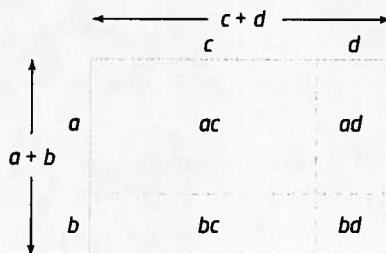
$$\begin{aligned}&= x(x + 4) - 2(x + 4) \\ &= x^2 + 4x - 2x - 8 \\ &= x^2 + 2x - 8\end{aligned}$$

- c) Apply the distributive property mentally. Perform the multiplications indicated by the upper arrows and then the lower arrows. Then, simplify by collecting like terms.

$$\begin{aligned}(3x + 7)(x - 5) &= 3x^2 - 15x + 7x - 35 \\ &= 3x^2 - 8x - 35\end{aligned}$$

You can find the product of two binomials by multiplying each term in the first binomial by each term in the second binomial. If necessary, simplify by collecting like terms.

$$(a + b)(c + d) = ac + ad + bc + bd$$



### Example 3 Expand and Simplify

Expand and simplify.

a)  $-2(4x - 5)(7x - 6)$

b)  $2(x + 7)(x - 3) - (4x + 3)(2x - 1)$

#### Solution

a) First, find the product of the binomials. Then, multiply by  $-2$ .

$$\begin{aligned} & -2(4x - 5)(7x - 6) \\ &= -2(28x^2 - 24x - 35x + 30) \\ &= -2(28x^2 - 59x + 30) \\ &= -56x^2 + 118x - 60 \end{aligned}$$

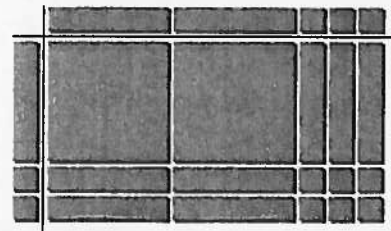
$$\begin{aligned} \text{b) } & 2(x + 7)(x - 3) - (4x + 3)(2x - 1) \\ &= 2(x^2 - 3x + 7x - 21) - (8x^2 - 4x + 6x - 3) \\ &= 2(x^2 + 4x - 21) - (8x^2 + 2x - 3) \\ &= 2x^2 + 8x - 42 - 8x^2 - 2x + 3 \\ &= -6x^2 + 6x - 39 \end{aligned}$$

Multiplying a polynomial by  $-1$  produces the opposite polynomial:  
 $(-1)(8x^2 + 2x - 3)$   
 $= -8x^2 - 2x + 3$

**Connections**  
 Expand, find the product, and multiply all mean the same thing.

#### Key Concepts

- You can model a binomial product as the area of a rectangle with the binomials as the dimensions.



The area of the rectangle is  $2x^2 + 7x + 6$ .  
 $(x + 2)(2x + 3) = 2x^2 + 7x + 6$

- You can find the product of two binomials by multiplying each term in one binomial by each term in the other binomial. If necessary, simplify by collecting like terms.

$$\begin{aligned} & (x + 2)(2x + 3) \\ &= 2x^2 + 3x + 4x + 6 \\ &= 2x^2 + 7x + 6 \end{aligned}$$

